

Simplifying and Graphing Rational Functions

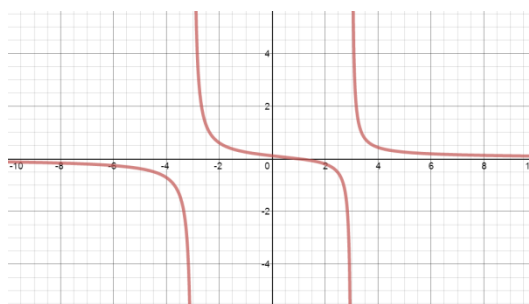
1. Pg 543 #11-19 odd and Pg 550 #11-19 odd
2. Pg 543 #12-18 even and Pg 550 #12-18 even
3. Worksheet
4. Worksheet
5. Pg 544 #25 and 34 and Pg 551 #23-35 column (need graph paper)
6. Pg 544 #28 and 37 and Pg 551 #24-36 column (need graph paper)
7. Pg 544 #31 and 40 and Pg 551 #25-37 column (need graph paper)
8. Worksheet
9. Worksheet
10. Chapter Review

Steps – Used for $f(x) = \frac{p(x)}{q(x)}$, where $p(x)$ and $q(x)$ are polynomials and $q(x) \neq 0$.

1. Write the rational function in factored form
2. Identify the domain of the function by setting the denominator factors to zero and solve
 - a. The domain restrictions will identify all of the vertical asymptotes and holes of the function
 - i. **Hole:** Any common factors between the numerator and denominator will yield a hole in the graph. Use this restriction as the x value and plug it into the simplified function to identify the ordered pair that represents the hole in the graph
 - ii. **Vertical Asymptote:** Any of the other restrictions on the domain that are not holes
3. Identify the degree of the numerator and the degree of the denominator
 - a. **Horizontal Asymptote at $y = 0$:** If the degree of the denominator is larger than the degree of the numerator
 - b. **Horizontal Asymptote at $y = \frac{\text{leading coefficient of numerator}}{\text{leading coefficient of denominator}}$:** If the degree of the numerator and denominator are the same
 - c. **Slant Asymptote (Oblique):** If the degree of the numerator is “1” degree larger than the degree of the denominator
 - i. The asymptote can be found by dividing the numerator of the rational function by the denominator of the rational function and ignoring the remainder. This can be done using long division or synthetic division.

* A rational function can only have a horizontal asymptote or a slant asymptote but not both*

Error Alert: The domain is determined by the vertical asymptotes but the range is not necessarily determined by the horizontal asymptotes. This is because vertical asymptotes cannot be crossed, but horizontal asymptotes can, in certain cases, be crossed. Horizontal asymptotes only look at the behavior of the graph as x gets really large or really small. The only easy way to determine the range of a function is to look at its graph.



Notice this graph has a horizontal asymptote at $y = 0$; however the portion of the graph found between the two vertical asymptotes would yield a range of all real numbers or $(-\infty, \infty)$.

Find the Domain, Vertical Asymptotes, Horizontal Asymptotes and Slant Asymptotes of the following rational functions.

E1. $y = \frac{x^2+3x+1}{4x^2-9}$

Holes:

Vertical Asymptotes:

Horizontal/Slant Asymptotes:

Domain (Interval Notation):

x-intercepts:

y-intercept:

E2. $y = \frac{x+2}{x^2+2x-8}$

Holes:

Vertical Asymptotes:

Horizontal/Slant Asymptotes:

Domain(Interval Notation):

x-intercepts:

y-intercept:

P1. $y = \frac{x^2+2x-3}{x^2-5x-6}$

Holes:

Vertical Asymptotes:

Horizontal/Slant Asymptotes:

Domain (Interval Notation):

x-intercepts:

y-intercept:

P2. $y = \frac{4x+1}{x^2-1}$

Holes:

Vertical Asymptotes:

Horizontal/Slant Asymptotes:

Domain(Interval Notation):

x-intercepts:

y-intercept:

$$E3. y = \frac{x^3-8}{x^2+9}$$

Holes:

Vertical Asymptotes:

Horizontal/Slant Asymptotes:

Domain(Interval Notation):

x-intercepts:

y-intercept:

$$E4. y = \frac{2x^3+4x^2-9}{3-x^2}$$

Holes:

Vertical Asymptotes:

Horizontal/Slant Asymptotes:

Domain(Interval Notation):

x-intercepts:

y-intercept:

$$P3. y = \frac{-x^3}{x^2+9}$$

Holes:

Vertical Asymptotes:

Horizontal/Slant Asymptotes:

Domain(Interval Notation):

x-intercepts:

y-intercept:

$$P4. y = \frac{x^3-8}{x^2+5x+6}$$

Holes:

Vertical Asymptotes:

Horizontal/Slant Asymptotes:

Domain(Interval Notation):

x-intercepts:

y-intercept:

$$E5. y = \frac{x^2 - x - 2}{x - 2}$$

Holes:

Vertical Asymptotes:

Horizontal/Slant Asymptotes:

Domain(Interval Notation):

x-intercepts:

y-intercept:

$$E6. y = \frac{x+3}{x^2+9}$$

Holes:

Vertical Asymptotes:

Horizontal/Slant Asymptotes:

Domain(Interval Notation):

x-intercepts:

y-intercept:

$$P5. y = \frac{x^2 + 7x + 12}{x^2 - 16}$$

Holes:

Vertical Asymptotes:

Horizontal/Slant Asymptotes:

Domain(Interval Notation):

x-intercepts:

y-intercept:

$$P6. y = \frac{x+2}{x^2+1}$$

Holes:

Vertical Asymptotes:

Horizontal/Slant Asymptotes:

Domain(Interval Notation):

x-intercepts:

y-intercept:

Steps – Used for a rational function in the form $y = \frac{a}{x-h} + k$

1. Use transformations to identify the vertical and horizontal asymptote
 - a. $x = h$ and $y = k$ are the asymptotes
2. Use the vertical asymptote to identify the domain of the function

E7. $y = \frac{-2}{x+3} - 2$

P7. $y = \frac{3}{x-1} + 2$

Holes:

Holes:

Vertical Asymptotes:

Vertical Asymptotes:

Horizontal/Slant Asymptotes:

Horizontal/Slant Asymptotes:

Domain(Interval Notation):

Domain(Interval Notation):

x-intercepts:

x-intercepts:

y-intercept:

y-intercept:

Steps

1. Factor the function
2. Find the intercepts, if there are any
 - a. x-intercepts: let $y = 0$ (Set the numerator to 0 and solve)
3. Find the vertical asymptotes, if any, and draw with a dotted line on the graph
4. Find the horizontal asymptote, if any, and draw with a dotted line on the graph
5. The vertical asymptotes will divide the x-axis into regions. Create a t-table choosing values between and beyond the vertical asymptotes
6. Plot the points, including the intercepts and sketch the graph

E1. Graph: $y = \frac{4}{x^2+1}$

Holes:

Vertical Asymptotes:

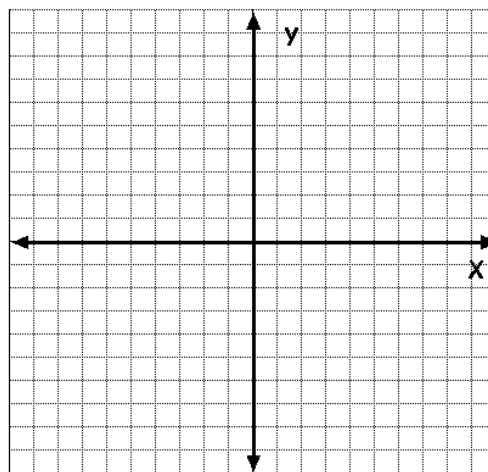
Horizontal/Slant Asymptotes:

Domain(Interval Notation):

x-intercept:

y-intercept:

Range (Interval Notation)



P1. Graph: $y = \frac{x}{x^2+1}$

Holes:

Vertical Asymptotes:

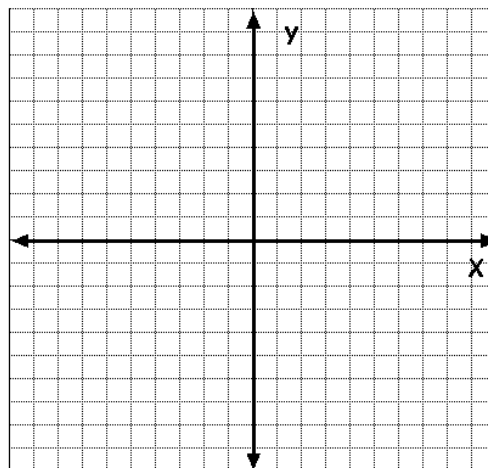
Horizontal/Slant Asymptotes:

Domain(Interval Notation):

x-intercept:

y-intercept:

Range (Interval Notation):



E2. Graph: $y = \frac{3x^2}{x^2-4}$

Holes:

Vertical Asymptotes:

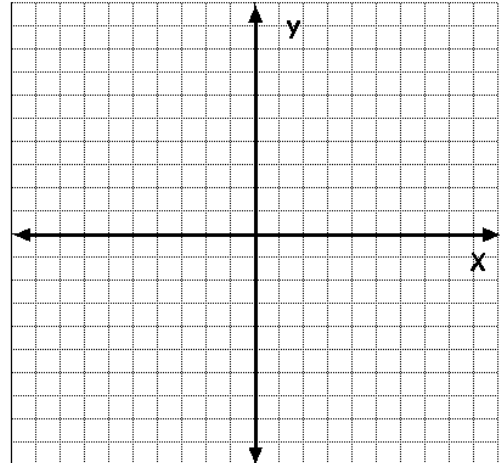
Horizontal/Slant Asymptotes:

Domain(Interval Notation):

x-intercept:

y-intercept:

Range (Interval Notation):



P2. Graph: $y = \frac{2x^3}{x^3-1}$

Holes:

Vertical Asymptotes:

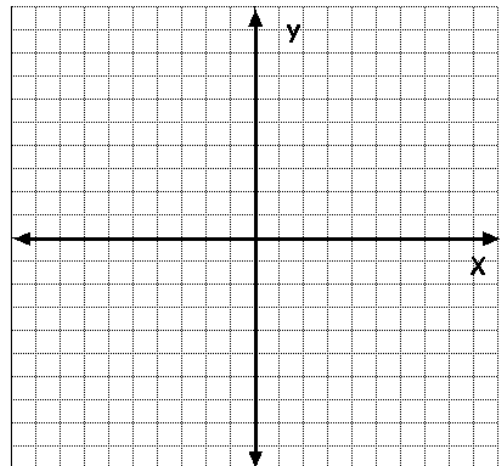
Horizontal/Slant Asymptotes:

Domain(Interval Notation):

x-intercept:

y-intercept:

Range (Interval Notation):



E3. Graph: $y = \frac{x^2 - 2x - 3}{x + 4}$

Holes:

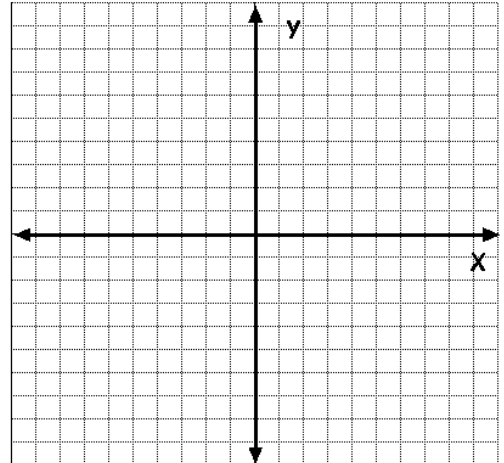
Vertical Asymptotes:

Horizontal/Slant Asymptotes:

Domain (Interval Notation):

x-intercept:

y-intercept:



P3. Graph: $y = \frac{x^2 - 3x - 4}{x - 2}$

Holes:

Vertical Asymptotes:

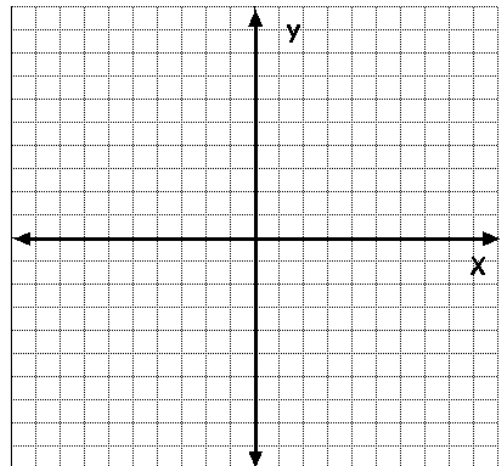
Horizontal/Slant Asymptotes:

Domain (Interval Notation):

x-intercept:

y-intercept:

Range (Interval Notation):



E4. Graph: $y = \frac{x+1}{2x-4}$

Holes:

Vertical Asymptotes:

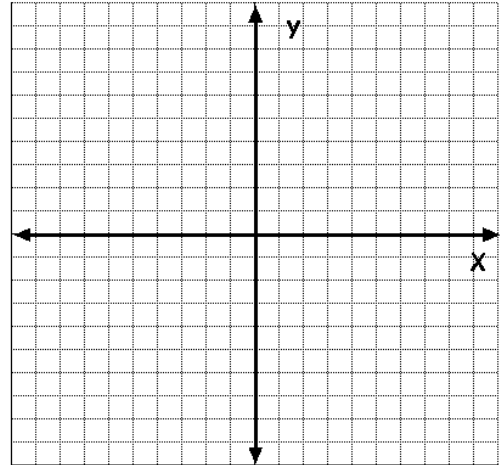
Horizontal/Slant Asymptotes:

Domain(Interval Notation):

x-intercept:

y-intercept:

Range (Interval Notation):



P4. Graph: $y = \frac{x-2}{3x+3}$

Holes:

Vertical Asymptotes:

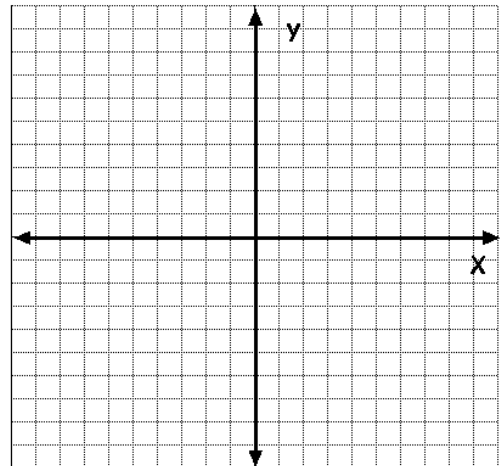
Horizontal/Slant Asymptotes:

Domain(Interval Notation):

x-intercept:

y-intercept:

Range (Interval Notation):



E5. Graph: $y = \frac{-2}{x+3} - 1$

Holes:

Vertical Asymptotes:

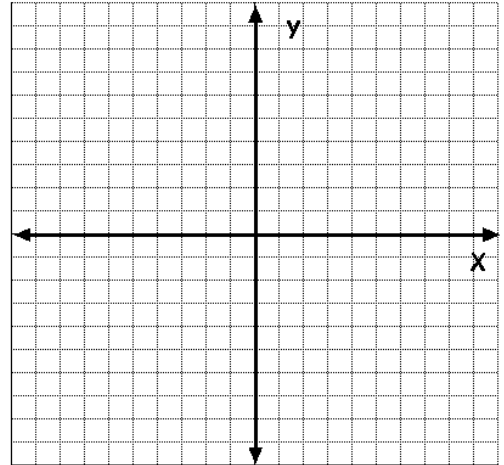
Horizontal/Slant Asymptotes:

Domain(Interval Notation):

x-intercept:

y-intercept:

Range (Interval Notation):



P5. Graph: $y = \frac{3}{x-1} + 2$

Holes:

Vertical Asymptotes:

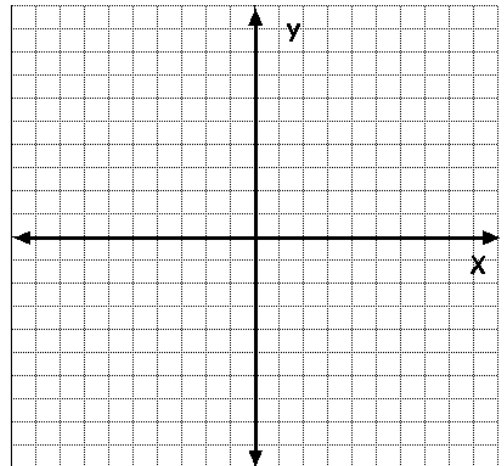
Horizontal/Slant Asymptotes:

Domain(Interval Notation):

x-intercept:

y-intercept:

Range (Interval Notation):



Warm-ups

Use the provided spaces to complete any warm-up problem or activity

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Warm-ups

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